

Graduate Research Plan Statement

Introduction/Background: Partial differential equations (PDEs) form the basis for modeling and controlling complex systems, governing how states evolve over time and space. From autonomous motion planning to financial modeling, PDEs describes the dynamics and optimality conditions that dictate safe and desirable behaviors. However, due to the computational cost of numerical methods, solving such PDEs remain intractable in high-dimensional, non-linear, or stochastic settings. These limitations remain as major hurdles across many domains, particularly in safety-critical and optimal controls applications.

In recent years, advancements in machine learning have opened new avenues for solving PDEs. Current deep learning PDE solvers are split in two directions: operator learning [1], which learns mappings between function spaces using data, and physics-constrained methods [2,3,4], which approximate solutions as neural networks by embedding physical laws directly into the training objective. The latter method, exemplified by physics-informed neural networks (PINNs) [2] and backwards stochastic differential equation (BSDE)-based solvers [3,4], shows potential in addressing the failures of traditional solvers in complex high-dimensional regimes as they do not require simulation data or explicit discretization. However, despite early successes, PINNs and BSDE-based methods face many well-documented failures.

PINNs utilizes a PDE-based residual loss to model the solution to boundary value problems. However, training PINNs has shown to be non-trivial, often requiring methods and tuning specific to each problem. Strategies such as adaptive loss weighing, adaptive activation functions, PDE regularization, residual-based loss resampling, and second-order optimization have been proposed to address its shortcomings. However, these strategies often fail to model complex solutions accurately, and, due to the curse of dimensionality, scaling PINNs to high-dimensional problems has yet to be fully realized.

Conversely, BSDE-based methods reformulate PDEs as BSDEs and optimize on a trajectory-based self-consistency loss. Using these methods, BSDE-based solvers have been successfully applied to various fields, such as financial derivatives pricing, risk management, and optimal control. However, existing studies have shown that BSDE-based methods often perform non-trivially worse than PINNs on empirical benchmarks.

Research Plan: I plan to address these challenges by developing a robust deep PDE solver which combines strategies from PINNs and BSDE-based methods to solve complex high-dimensional PDE problems naturally arising from control theory. Initial work has been conducted under Dr. Stephen Tu at USC, investigating methods of incorporating spatio-temporal dependencies in PINNs and the failures of BSDE-based solvers. Specifically in [5], we identify a non-trivial bias stemming from the standard Euler-Maruyama (EM) discretization used in BSDE-based solvers which persists even with finer discretizations (cf. Figure 2). We propose a Heun-based BSDE scheme which eliminates the bias and restores parity with and often outperforms PINNs on high-dimensional benchmarks. Building on this work, I propose three distinct research thrusts: **1. Further investigate strategies to improve PINNs and BSDE performance in high-dimensional problems, 2. Apply these strategies to scale safe and optimal control to complex, high-dimensional systems, 3. Extend physics-constrained learning towards world models and embodied AI.**

Thrust 1: While it has been shown that PINNs can produce highly accurate solutions, PINNs often fail to converge well due to the ill-conditioned optimization landscape [6]. In contrast, many existing strategies focus on modifying the network architecture or loss/PDE formulation. Building on the idea of an ill-conditioned loss, **I believe that loss smoothing and higher-order optimization frameworks that leverage the PDE structure in PINNs can improve performance without increasing training complexity.**

Conversely, it is hypothesized that BSDE-based loss landscapes are inherently smoother than that of PINNs due to its stochastic and trajectory-based nature. Despite these advantages, BSDE-based methods

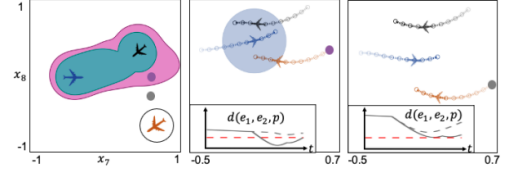


Figure 1: Unsafe set boundary for airplane collisions. Reproduced from [8].

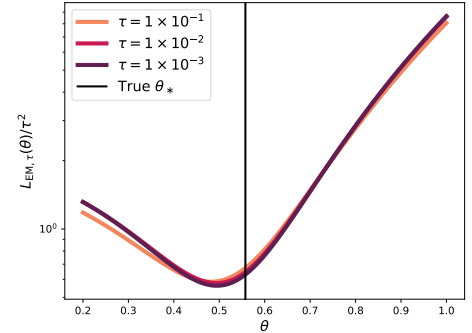


Figure 2: A plot of the EM-BSDE loss at various levels of discretization on a 1D linear quadratic regulator HJB problem. Reproduced from [5].

have not received nearly as much attention as PINNs in recent literature. **By investigating higher-order integration schemes, as well as incorporating PINNs-based model and optimization strategies, I plan to improve the SoTA accuracy achievable by BSDE-based methods.**

Thrust 2: Recently, Hamilton-Jacobi (HJ)-reachability has emerged as a principled method for deriving formal safety guarantees in dynamical systems using HJ-PDEs. Control barrier value functions (CBVFs) [7] aims to unify HJ-reachability with control barrier functions (CBFs), providing a principled method for generating barrier functions and enabling verifiably safe controls in real-time. While CBFs are effective for ensuring safety and performance, existing methods for solving HJ-reachability [8] rely on PINNs which suffers from poor sampling and optimization issues, limiting these methods to low-dimensional deterministic systems. I plan to extend HJ-reachability and CBVFs to high-dimensional stochastic systems using the improved BSDE-based methods from Thrust 1, which are better suited for high-dimensional settings. Since BSDEs provide a natural stochastic representation of HJ PDEs, this approach will enable efficient sampling and provide a scalable and consistent method for computing CBVFs. **Through this effort, I aim to demonstrate the application of deep BSDE methods to safety-critical control and establish a foundation for learning-based controllers with formal safety guarantees.**

Thrust 3: World models provide a compact latent representation of an environment’s dynamics, allowing for efficient decision making without complex full-scale simulations. Building upon the robust deep PDE solver and its application to CBVFs developed in previous thrusts, I aim to generalize these methods towards scalable physics-grounded representations and decision making in dynamic environments. By combining physics-constrained methods with latent representations in world models, I seek to develop physically consistent predictive models that can capture system dynamics for safe and adaptive control. Specifically, PDE-learning offers a principled alternative in domains such as fluid and flight dynamics, structural response, and multi-physics interactions where physical consistency and accurate long-horizon predictions remain challenging for data-driven methods. The development of such predictive world models align with the broader vision of embodied AI, where agents reason through learned models of their environment for effective decision-making. Overall, **I hope to establish the framework for integrating deep PDE learning with predictive modeling in complex systems, enabling real-time planning, adaptation, and safety in high-dimensional, uncertain environments.**

Intellectual Merit:

The proposed research aims to bridge the gaps in the current scientific machine learning (SciML) landscape by unifying PINNs and BSDE-based solvers to develop a robust deep PDE solver in complex high-dimensional systems. In addition, the effective application of deep PDE solvers to optimal control and physically consistent predictive modeling will provide novel insights into PDE learning based methods for control-theoretic applications and demonstrate the potential for deep PDE solvers to address real-complex problems. If successful, this work will advance the state-of-the-art for solving high-dimensional PDEs in domains such as control theory, which remains intractable using current methods.

Broader Impacts:

By scaling safe and optimal controls to high-dimensional stochastic systems, this work will directly contribute to safer autonomy across transportation, service, and industrial infrastructure. Additionally, the proposed research will provide a foundation for robust and efficient decision-making, significantly improving overall quality of life by integrating robotics and autonomy to greater society. Outside of controls, PDEs, as the backbones of physics and engineering, are responsible for modeling various complex phenomena in nature. From better weather and earthquake predictions to advancements in fields such as plasma physics, superconductors, and medical imaging, a robust and efficient method for solving PDEs stands to improve our understanding of the world and catalyze scientific breakthroughs across many fields, benefiting both the STEM community and society at large.

To aid the potential of deep PDE solvers to revolutionize science, findings from this research, both positive and negative, will be shared with the broader community, and all developed code and models will be made publicly available. Additionally, by proving the effectiveness of deep PDE solvers and collaborating with domain experts at USC across areas ranging from distributed drone systems to quantum mechanics, I aim to contribute to interdisciplinary research in machine learning and the wider scientific community.

References: [1] N. Kovachki et al., JMLR, 2024. [2] M. Raissi et al., JCP, 2019. [3] M. Raissi, 2018, arXiv:1804.07010. [4] J. Han et al., PNAS, 2018. [5] S. Park et al., NeurIPS, 2025. [6] P. Rathore et al., ICML, 2024. [7] J. Choi et al., CDC, 2021. [8] S. Bansal et al., ICRA 2021.